Bressoud's Conjecture on the Rogers-Ramanujan Identities

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Abstract

In 1980, Bressoud put forward a conjecture for a more general partition identity that implies many classical results in the theory of partitions, such as, Euler's partition theorem, the Rogers-Ramanujan-Gordon identities and the Andrews-Göllnitz-Gordon identities and so on. Bressoud's conjecture depends on several parameters, and here we simply stated as $A_j(\alpha_1,\ldots,\alpha_\lambda;\eta,k,r;n)=B_j(\alpha_1,\ldots,\alpha_\lambda;\eta,k,r;n)$ for $j=0$ or 1, where the function $A_j(\alpha_1,\ldots,\alpha_\lambda;\eta,k,r;n)$ counts the number of partitions with certain congruence conditions and the function $B_j(\alpha_1,\ldots,\alpha_\lambda;\eta,k,r;n)$ counts the number of partitions with certain difference conditions. Bressoud's conjecture was known in some special cases. The general case for $j=1$ was recently resolved by Kim.

In this talk, we present an answer to Bressoud's conjecture for the case $j=0$, resulting in a complete solution to the conjecture. It is somewhat unexpected that overpartitions play a crucial role in this regards. We first introduce a new partition function $\overline{B}_j(\alpha_1,\ldots,\alpha_\lambda;\eta,k,r;n)$ which could be viewed as an overpartition analogue of the partition function $B_j(\alpha_1,\ldots,\alpha_\lambda;\eta,k,r;n)$ introduced by Bressoud. By constructing bijections, we show that there is a relationship between $\overline{B}_1(\alpha_1,\ldots,\alpha_\lambda;\eta,k,r;n)$ and $B_0(\alpha_1,\ldots,\alpha_\lambda;\eta,k,r;n)$ and a relationship between $\overline{B}_0(\alpha_1,\ldots,\alpha_\lambda;\eta,k,r;n)$ and $B_1(\alpha_1,\ldots,\alpha_\lambda;\eta,k,r;n)$. Based on these two relations, we confirm Bressoud's conjecture for $j=0$. Besides, our approach leads to overpartition analogues of Bressoud's conjecture, which cover a number of overpartition analogues of classical theorems in the theory of partitions. The generating functions of overpartition analogues of Bressoud's conjecture are also obtained with the aid of Bailey pairs. The overpartition analogues explored in this study are not merely a matter of expansion or refinement, but rather are a crucial and fundamental component in our solution to Bressoud's conjecture regarding ordinary partitions. This is joint work with Thomas Y. He and Alice X.H. Zhao.