When I turned 40 I was a professional musician and former college drop-out; I decided to drop out of music to study mathematics. I returned to college and earned my B.S. and Ph.D (Emory University, 2018). I will turn 50 on Frank Garvan's Birthday (March 9), so this will be the last talk of a decade that transformed my life profoundly, in large part thanks to members of this audience. I will give an incomplete survey of a meta-theory of partitions I began developing in graduate school with my first paper on partition zeta functions (2016), that is still under construction by myself and my collaborators (Akande, Dawsey, Hendon, Jameson, Just, Ono, Pulagam, Rolen, M. Schneider, Sellers, Sills, Wagner), as well as my UGA undergraduate research groups, and other authors. I hope to follow with talks on specific topics in future sessions; this talk will lay the notational and conceptual groundwork.

Much like the natural numbers $\mathbb{N}$, the set $\mathcal{P}$ of integer partitions ripples with interesting patterns and relations. Now, Euler's product formula for the zeta function as well as his generating function formula for the partition function $p(n)$ share a common theme, despite their analytic dissimilarity: expand a product of geometric series, collect terms and exploit arithmetic structures in the terms of the resulting series. Work of Alladi-Erdős and Andrews hint at algebraic and analytic super-structures unifying aspects of partition theory and arithmetic. One wonders then: might some theorems of classical multiplicative number theory arise as images in $\mathbb{N}$ of greater algebraic or set-theoretic structures in $\mathcal{P}$?

We show that many well-known objects from elementary and analytic number theory can be viewed as special cases of phenomena in partition theory such as: a multiplicative arithmetic of partitions that specializes to many theorems of elementary number theory; a class of "partition zeta functions" containing the Riemann zeta function and other Dirichlet series (as well as exotic non-classical cases); partition-theoretic methods to compute arithmetic densities of subsets of $\mathbb{N}$ as limiting cases of q-series; and other phenomena at the intersection of the additive and multiplicative branches of number theory.